Stofftransport in der Fließstrecke

$$\frac{\partial c}{\partial t} = -div(\vec{u}c) + \cancel{div(D\,grad(c))} + \cancel{r}$$

Hydrodynamik im PFR

Dispersionsmodell (z-Richtung)

$$\frac{\partial c}{\partial t} = -u_z \frac{\partial c}{\partial z} + D\frac{\partial^2 c}{\partial z^2}$$

Für die Dispersion beidseitig offenes System

z=0 z=L

Analytisch lösbar für δ-Signal

Für die Dispersion beidseitig geschlossenes System

Analytisch nicht lösbar

Stofftransport in der Fließstrecke

Dispersionsmodell (z-Richtung)

$$\frac{\partial c}{\partial t} = -u_z \frac{\partial c}{\partial z} + D\frac{\partial^2 c}{\partial z^2}$$

Pulsförmige Eingangssignal (δ) beidseitig offen

$$c(t,z) = \frac{n}{2A\sqrt{\pi\, D_{ax} t}}\, e^{-\left(\frac{(z-u_z t)^2}{4D_{ax} t}\right)} \quad \text{mit} \quad \frac{n}{A} = \int_{-\infty}^{\infty} c(t,z)\,dz$$

für A=1, D_{ax}=1, u_z=1, z=0..100, t=2..100

Stofftransport in der Fließstrecke

Dispersionsmodell

$$\frac{\partial c}{\partial t} = -u_z \frac{\partial c}{\partial z} + D\frac{\partial^2 c}{\partial z^2}$$

Einführung dimensionsloser Größen

$$x = \frac{z}{L} \qquad \theta = \frac{t}{\tau} \qquad \tau = \frac{L}{u_z}$$

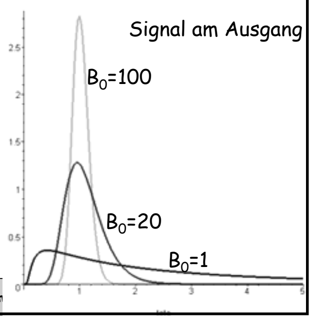
$$\frac{\partial c}{\partial \theta} = -\frac{\partial c}{\partial x} + \frac{D}{u_z L}\frac{\partial^2 c}{\partial x^2}$$

$$B_o = \frac{u_z L}{D} \quad \text{Bodensteinzahl}$$

Eingang δ; Ausgang:

$$\frac{c(\theta, B_o)}{c_0} = \frac{1}{2}\sqrt{\frac{B_o}{\pi\,\theta}}\, e^{-\left(\frac{(1-\theta)^2 B_o}{4\theta}\right)}$$

Signal am Ausgang



Bitte berechnen Sie die Halbwertsbreite der Gaußverteilung

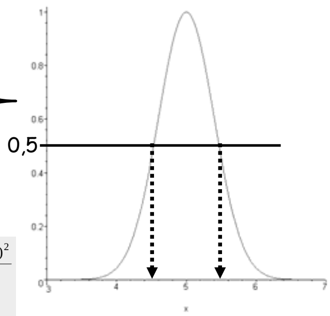
$$G(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\mu = 5$$

$$\sigma = \frac{1}{\sqrt{2\pi}} \quad \sigma=0{,}399$$

$$G(x_{0,5}) = \frac{1}{2}\frac{1}{\sigma\sqrt{2\pi}}$$

$$\frac{1}{2\sigma\sqrt{2\pi}} = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x_{0,5}-\mu)^2}{2\sigma^2}}$$



$$\frac{1}{2\sigma\sqrt{2\pi}} = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x_{0,5}-\mu)^2}{2\sigma^2}} \qquad \frac{1}{2} = e^{-\frac{(x_{0,5}-\mu)^2}{2\sigma^2}}$$

$$\ln\left(\frac{1}{2}\right) = -\frac{(x_{0,5}-\mu)^2}{2\sigma^2} \qquad \ln(1) - \ln(2) = -\frac{(x_{0,5}-\mu)^2}{2\sigma^2}$$

$$\ln(2) = \frac{(x_{0,5}-\mu)^2}{2\sigma^2} \qquad 2\sigma^2\ln(2) = (x_{0,5}-\mu)^2$$

$$\sqrt{2\sigma^2\ln(2)} = \pm(x_{0,5}-\mu) \qquad x_{0,5} = \mu \pm \sqrt{2\sigma^2\ln(2)}$$

$$x_{0,5} = \mu \pm \sigma\sqrt{2\ln(2)} \qquad \sqrt{2\ln(2)} = 1{,}177$$

$$x_{0,5} = \mu \pm 1{,}177\sigma$$

$$G(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$x_{0,5} = \mu \pm \sigma\sqrt{2\ln(2)}$$

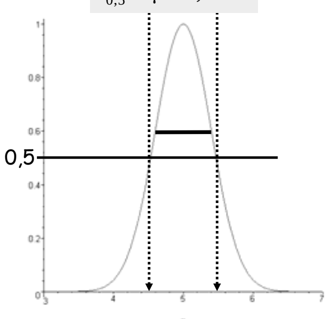
$$x_{0,5} = \mu \pm 1{,}177\sigma$$

für $\sigma = \frac{1}{\sqrt{2\pi}}$ gilt

$$G(\mu \pm \sigma) = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

$$G(\mu \pm \sigma) = \frac{1}{\sqrt{2{,}718}}$$

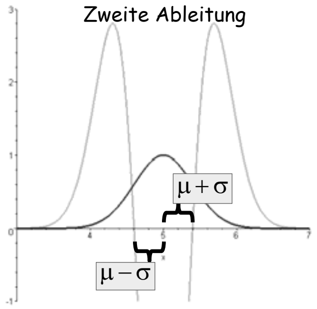
$$G(\mu \pm \sigma) = 0{,}6065$$



$$G(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$x_{0,5}=\mu\pm\sigma\sqrt{2\ln(2)}$$

$$x_{0,5}=\mu\pm 1{,}177\sigma$$



Wendepunkt

$$G''(x_w)=0$$

$$\longrightarrow\quad x_w=\mu\pm\sigma$$

$$\int_{\mu-\sigma}^{\mu+\sigma}\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx=0{,}683$$

Gegeben ist folgende Verweilzeitverteilung. Bitte berechnen Sie die Varianz dieser Verteilung!

$$E(t)=\frac{e^{-\frac{t}{\tau}}}{\tau}$$

Wahrscheinlichkeitsdichte-Verteilung eines idealen CSTR

$$\mathrm{var}(t)=\int_0^\infty (t-\bar{t})^2E(t)dt$$

2. Moment der Verteilung

$$\bar{t}=\int_0^\infty tE(t)dt$$

1. Moment der Verteilung

$$E(t)=\frac{e^{-\frac{t}{\tau}}}{\tau}\qquad \bar{t}=\int_0^\infty tE(t)dt\qquad \boxed{\int_0^\infty te^{-\frac{t}{\tau}}dt=\tau^2}$$

$$\bar{t}=\int_0^\infty t\frac{e^{-\frac{t}{\tau}}}{\tau}dt=-te^{-\frac{t}{\tau}}\Big|_0^\infty+\int_0^\infty e^{-\frac{t}{\tau}}dt=0-\tau e^{-\frac{t}{\tau}}\Big|_0^\infty=\tau$$

$$\int_a^b uv'\,dt=uv|_a^b-\int_a^b u'vdt\qquad u=t\quad u'=1\quad v=-e^{-\frac{t}{\tau}}\quad v'=\frac{1}{\tau}e^{-\frac{t}{\tau}}$$

$$E(t)=\frac{e^{-\frac{t}{\tau}}}{\tau}\longrightarrow \bar{t}=\tau$$

$$E(t)=\frac{e^{-\frac{t}{\tau}}}{\tau}\qquad \bar{t}=\tau\qquad \mathrm{var}(t)=\int_0^\infty(t-\bar{t})^2E(t)dt\qquad \boxed{\int_0^\infty te^{-\frac{t}{\tau}}dt=\tau^2}$$

$$\mathrm{var}(t)=\int_0^\infty(t-\tau)^2\frac{e^{-\frac{t}{\tau}}}{\tau}dt=\int_0^\infty\left(t^2-2\tau t+\tau^2\right)\frac{e^{-\frac{t}{\tau}}}{\tau}dt$$

$$\mathrm{var}(t)=\frac{1}{\tau}\int_0^\infty t^2e^{-\frac{t}{\tau}}dt-2\int_0^\infty te^{-\frac{t}{\tau}}dt+\tau\int_0^\infty e^{-\frac{t}{\tau}}dt$$

$$\mathrm{var}(t)=\frac{1}{\tau}\int_0^\infty t^2e^{-\frac{t}{\tau}}dt-2\tau^2+\tau(-\tau)e^{-\frac{t}{\tau}}\Big|_0^\infty=\frac{1}{\tau}\int_0^\infty t^2e^{-\frac{t}{\tau}}dt-2\tau^2-\tau^2(0-1)$$

$$\mathrm{var}(t)=\frac{1}{\tau}\int_0^\infty t^2e^{-\frac{t}{\tau}}dt-\tau^2$$

$$\mathrm{var}(t)=\frac{1}{\tau}\underbrace{\int_0^\infty t^2e^{-\frac{t}{\tau}}dt}-\tau^2\qquad \boxed{\int_0^\infty te^{-\frac{t}{\tau}}dt=\tau^2}$$

$$\int_a^b uv'\,dt=uv|_a^b-\int_a^b u'vdt\qquad u=t^2\quad u'=2t\quad v=-\tau e^{-\frac{t}{\tau}}\quad v'=e^{-\frac{t}{\tau}}$$

$$\int_0^\infty t^2e^{-\frac{t}{\tau}}dt=-t^2\tau e^{-\frac{t}{\tau}}\Big|_0^\infty+\int_0^\infty 2t\,\tau e^{-\frac{t}{\tau}}dt=2\tau\int_0^\infty te^{-\frac{t}{\tau}}dt=2\tau^3$$

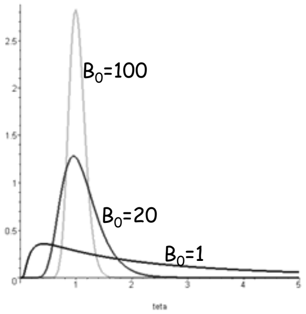
$$\mathrm{var}(t)=\frac{1}{\tau}2\tau^3-\tau^2$$

$$\mathrm{var}(t)=\tau^2$$

$$E(t)=\frac{e^{-\frac{t}{\tau}}}{\tau}\qquad \bar{t}=\tau \text{ 1. Moment}\qquad \mathrm{var}(t)=\tau^2 \text{ 2. Moment}$$

Bitte berechnen Sie die Varianz des Verweilzeitspektrums beim Dispersionsmodell

$$E(\theta)=\frac{1}{2}\sqrt{\frac{B_0}{\pi\theta}}e^{-\frac{B_0(1-\theta)^2}{4\theta}}$$



$$B_0=\frac{uL}{D_{ax}}$$

$$\bar{\theta}=\int_0^\infty\theta E(\theta)d\theta$$

$$\mathrm{var}(\theta)=\int_0^\infty(\theta-\bar{\theta})^2E(\theta)d\theta$$

Dispersionsmodell

```
> restart;
> E(theta):=sqrt(B0/Pi/theta)/2*exp(-B0*(1-theta)^2/4/theta);
```

$$E(\theta) := \frac{1}{2}\sqrt{\frac{B0}{\pi\,\theta}}\,e^{\left(-\frac{B0\,(1-\theta)^2}{4\,\theta}\right)}$$

$$E(\theta) = \frac{1}{2}\sqrt{\frac{B_0}{\pi\theta}}\,e^{-\frac{B_0(1-\theta)^2}{4\theta}}$$

```
> assume(B0>0);
>
>
> |
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$$\overline{\theta} = \int_0^\infty \theta E(\theta)\,d\theta = 1 + \frac{2}{B_0}$$

$$\mathrm{var}(\theta) = \int_0^\infty (\theta - \overline{\theta})^2 E(\theta)\,d\theta = \frac{2}{B_0^2}\left(B_0 + 4\right)$$